USQCD US Lattice Quantum Chromodynamics

Heavy Quark Flavor Physics from Lattice QCD

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(University of Illinois)

Lattice meets Experiment

Fermilab

07 March 2014

Outline

- Introduction
- Leptonic and SL decays
  - D mesons
  - B mesons
- Neutral meson mixing
  - B mesons
  - D mesons
- Conclusion & Outlook

Lattice Averages

We have independent lattice results from different lattice groups using different methods for an increasing number of quantities
⇒ need averages ⇒ inputs into UT fits

- two efforts:

1. FLAG -1 (**F**lavianet **L**attice **A**veraging **G**roup)
   Colangelo, et al (Eur. Phys. J. C71 (2011) 1695, http://itpwiki.unibe.ch/flag/) 12 people (EU)
   light quark quantities only

2. LLV (**L**aiho, **L**unghi, **V**an de Water)
   (Phys.Rev.D81:034503,2010, http://latticeaverages.org/)
   light and heavy quark quantities

   + UT fits with lattice averages as input

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= FLAG -2 (**F**lavor **L**attice **A**veraging **G**roup) http://itpwiki.unibe.ch/flag/

28 people (EU, US, Japan) representing the big lattice collaborations

light and heavy quark quantities

1^{st} review (arXiv:1310.8555 with April 30 deadline → revision in progress with November 30 deadline)

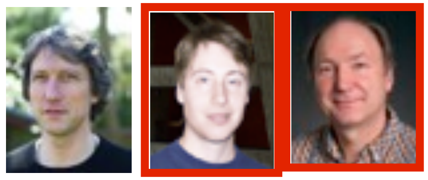
The FLAG-2 collaboration

Editorial Board

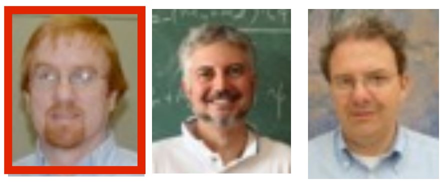
Advisory Board

Vus and Vud

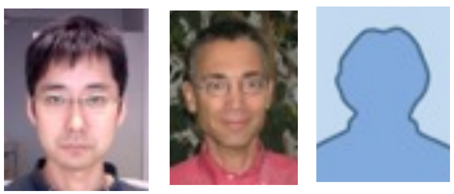
Kaon B-parameter



Light Quark Masses

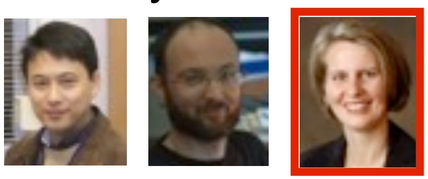


LECs

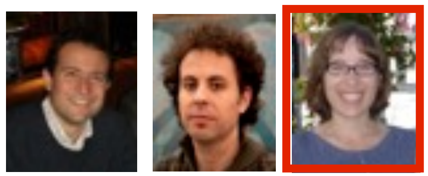


Strong Coupling

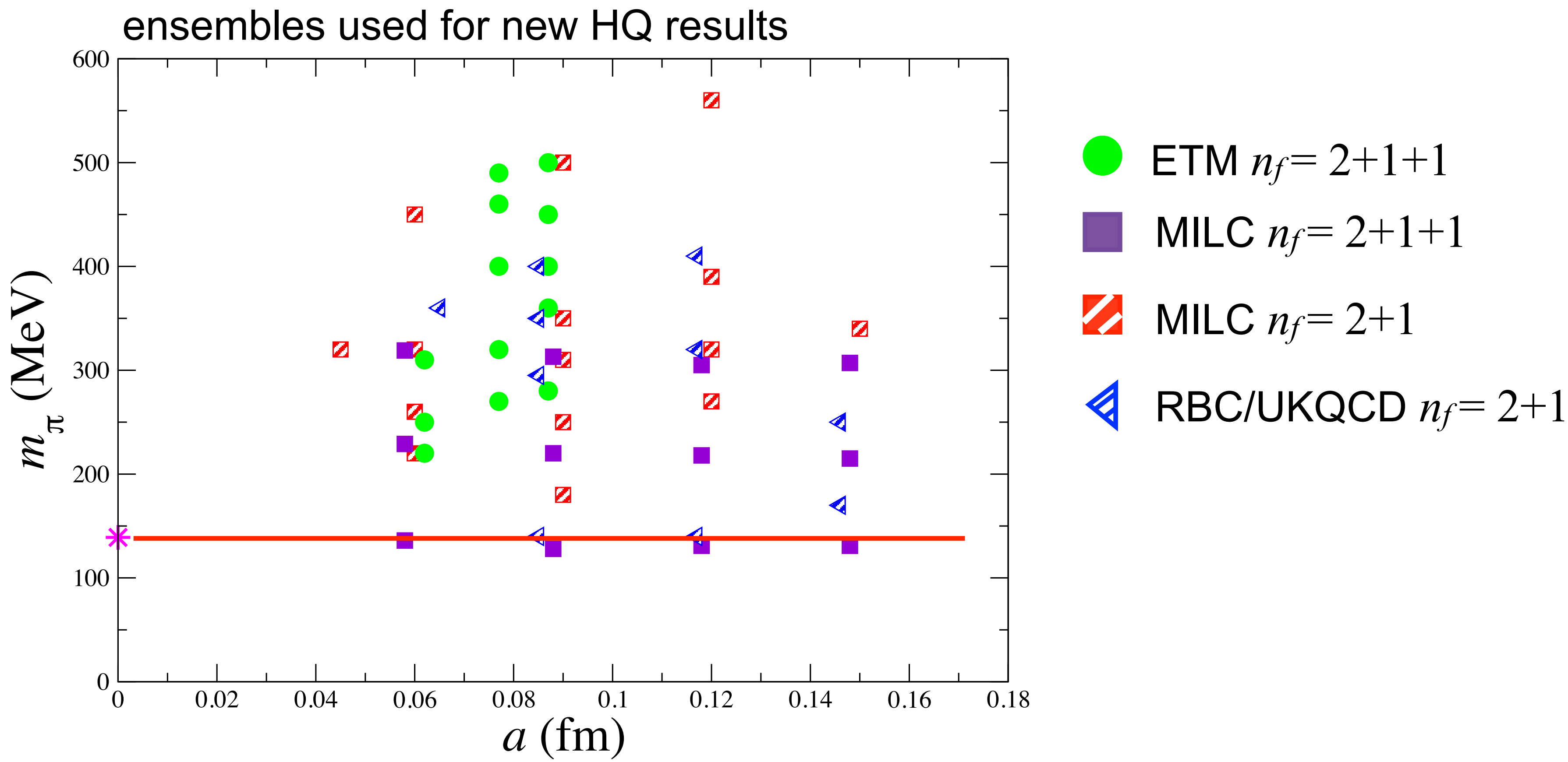
B,*D* decay constants and *B* mixing



Semileptonic *B*,*D* meson form factors



Ensemble overview



MILC used by FNAL/MILC, HPQCD, SWME
RBC/UKQCD used by χQCD
ETM used by Orsay

Heavy Quark Treatment

- For light quarks ($m_\ell < \Lambda_{\rm QCD}$), discretization errors ~ $\alpha_s^k (a\Lambda_{\rm QCD})^n$
- For heavy quarks, discretization errors ~ $\alpha_s^k (am_h)^n$
  with currently available lattice spacings
  - for b quarks $am_b > 1$
  - for charm $am_c \sim 0.15\text{-}0.6$

$\Rightarrow$ need effective field theory methods for *b* quarks
for charm can use light quark methods, if action is sufficiently improved

- avoid errors of $(am_b)^n$ in by using EFT:
  - ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba)
  - ✦ HQET
  - ✦ NRQCD

or

- use improved light quark actions for charm (HISQ, tmWilson, NP imp. Wilson, Overlap, ...) and for *b*:
  - ✦ use same LQ action as for charm but keep $am_h < 1$,
  - ✦ use HQET and/or static limit to extrapolate/interpolate to *b* quark mass

Heavy Quark Treatment

Relativistic Heavy Quarks
(Fermilab, Columbia, Tsukuba)

- start with a relativistic action, usually Wilson + $O(a)$ improvement
- with mass-dependent matching conditions, cut-off effects are

$$\alpha_s^k f(m_h a)(a\Lambda)^n \quad \text{with}$$

$$am_h \sim 1 : f(m_h a) \sim O(1)$$

current implementations:

FNAL/MILC: tree-level tadpole $O(a)$ improved + 1-loop PT

PACS-CS: tree-level $O(a)$ improved : NP @ m_h=0 + 1-loop PT for m_h dependence

RBC: NP $O(a)$ improved

Heavy Quark Treatment

NRQCD:

- expansion in v^2 or Λ/m_h and a
- continuum limit defined as $a \to 0$ does not exist
  (power-law divergent terms in coefficients)
- keep $m_h a > 1$, i.e. a finite.
- "continuum limit" at $a \neq 0$:
  match and improve to high enough order in Λ/m_h and a
  so that residual truncation and discretization errors are small
- needs a scaling window where $a\Lambda \ll 1$ and $m_h a > 1$

  for b quarks $\Rightarrow$ $a \gtrsim 0.05$ fm

Current implementation (HPQCD):

- errors $\sim O(\alpha_s v^2), O(\alpha_s v^4), O(v^6), O(\alpha_s (a\Lambda)^2)$

Heavy Quark Treatment

HQET:

- leading term, static limit:
  - $O(a)$ improved, $1/m_Q$ effects not included
  - SU(3) breaking ratios have suppressed truncation errors

- systematic expansion in $1/m_Q$
- matching NP to obtain $1/m_Q$ accuracy

current implementations:

HQET (ALPHA): NP matched through $1/m_Q$ + NPR
error $\sim O(a\Lambda^2/m_h, (\Lambda/m_h)^2, (a\Lambda)^2)$

static (RBC/UKQCD): for SU(3) breaking ratios,
$1/m_Q$ error estimated by power counting

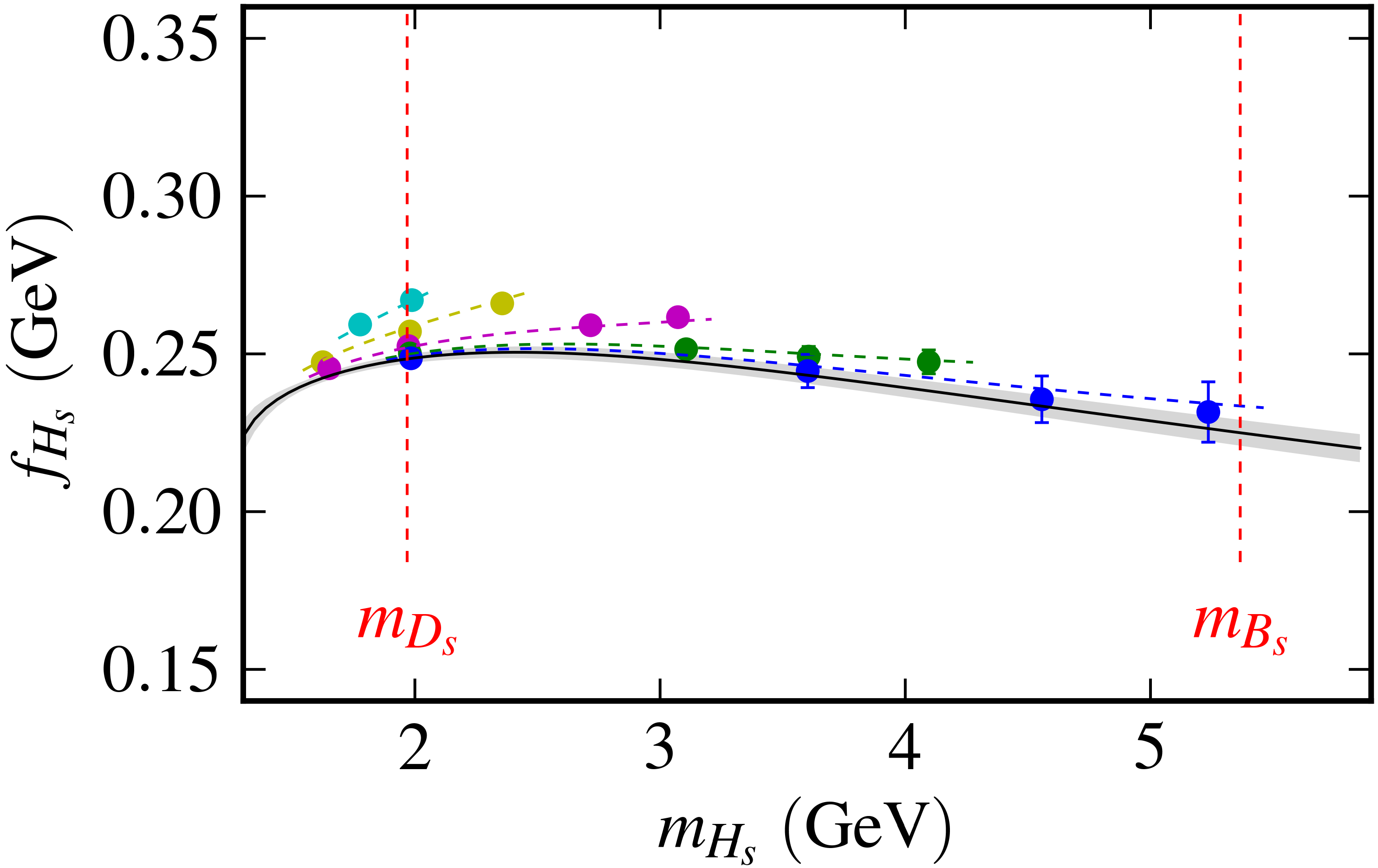
Heavy Quark Treatment

Light Quark Actions for b quarks

- Heavy HISQ method (HPQCD):
  - HISQ action is highly improved for heavy quarks:
    $$\sim \alpha_s \Lambda/m_h (am_h)^2, (\Lambda/m_h)^2 (am_h)^4$$
  - MILC ensembles with $a \approx 0.045 - 0.15$ fm
  - keep $am_h \leq 0.85$
  - extrapolate to b quark mass using HQET inspired expansion.

Heavy HISQ method

HPQCD 11A (Phys.Rev. D85 (2012) 031503)



HPQCD sees similar behavior for other HL quantities. For HH quantities discretization errors are a bit larger (but still small).

Heavy Quark Treatment

Light Quark Actions for b quarks

- Ratio Method (ETM):
  - use improved action with $am_h \leq 0.6$
  - construct ratios (z) of quantities so that $m_h \to \infty : z \to 1$
    For example, for

    $$f_B : \phi(m_h) \equiv f_{h\ell}\sqrt{m_H} \quad \to \quad z = \phi(m_h)/\phi(m_h/\lambda)$$

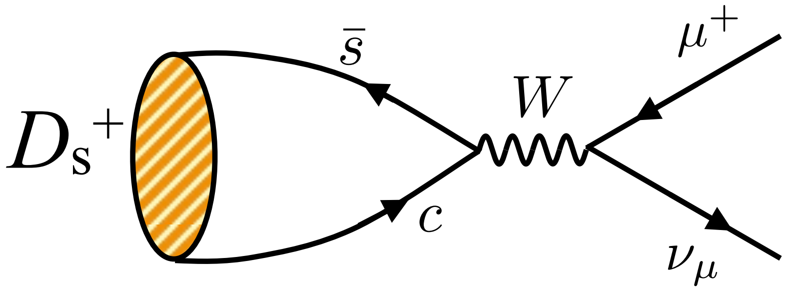
    where λ ~ 1.2
  - discretization errors are suppressed for such ratios
  - use HQET to extrapolate to physical b quark mass.

D meson physics

- leptonic $f_D, f_{D_s}, f_{D_s}/f_D$
- semileptonic $D \to K(\pi)\ell\nu$
- V_{cs} and V_{cd}

D and D_s meson decay constants

example: $D_s^+ \to \mu^+ \nu_\mu$



$$\Gamma(D_s^+ \to \mu^+ \nu_\mu) = (\text{known}) \times |V_{cs}|^2 \times f_{D_s^+}^2$$

- use experiment + LQCD input for determination of CKM element
- same for B ($|V_{ub}|$) meson
- experimental uncertainty (Rosner & Stone, arXiv:1309.1924):

  D_s: 1.8% D^+: 2.4%

  radiative correction of ~1% included for muon final state

FLAG-2 plot conventions

Satisfies all quality criteria; included in average.

Satisfies all quality criteria, but not included in average because the result is superseded or published in a conference proceedings.

Doesn’t satisfy all quality criteria; not included in average.

FLAG-2 average for each N_f

Non-lattice result

My ~~FLAG-2~~ plot conventions

New results. **Not rated** for FLAG-2 quality criteria; **not** included in average.

Satisfies all quality criteria; included in average.

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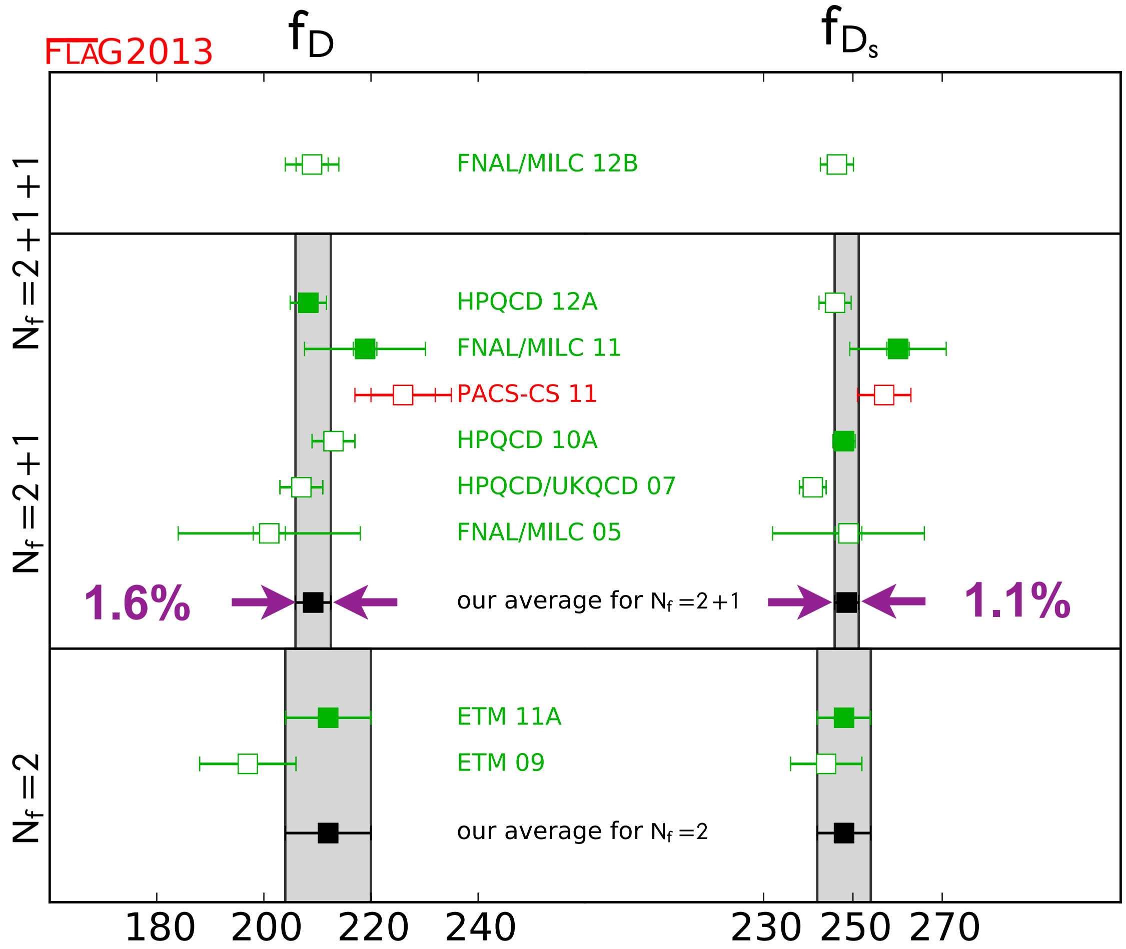
FLAG-2 average for each N_f

Non-lattice result

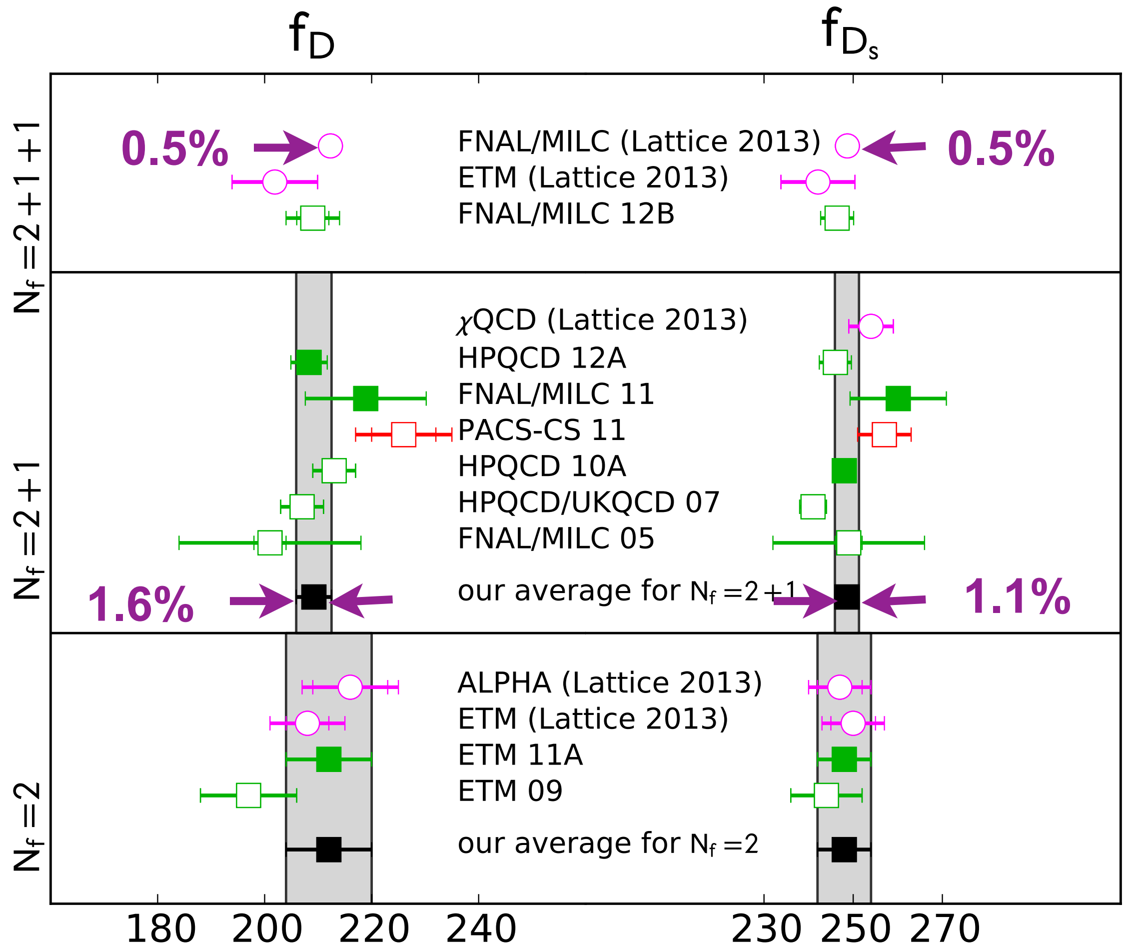
D and D_s meson decay constants



small errors due to

- ✦ highly improved action (HISQ)
- ✦ absolutely normalized current
- ✦ Asqtad ensembles with small lattice spacings

D and *D_s* meson decay constants



New results (shown in magenta) **not** included in FLAG-2 averages.

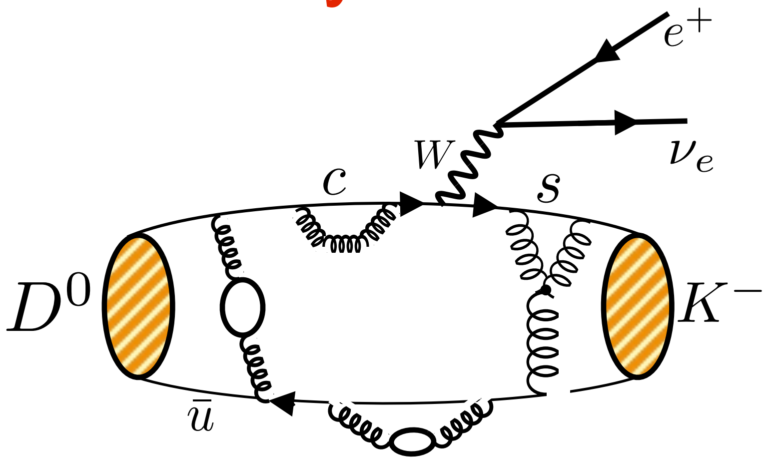
small errors due to

- ✦ **physical light quark masses**
- ✦ highly improved action (HISQ)
- ✦ absolutely normalized current
- ✦ HISQ ensembles with small lattice spacings (0.06 fm)

New results with other improved actions (DWF, twisted-mass Wilson, NP Wilson)

semileptonic D decay

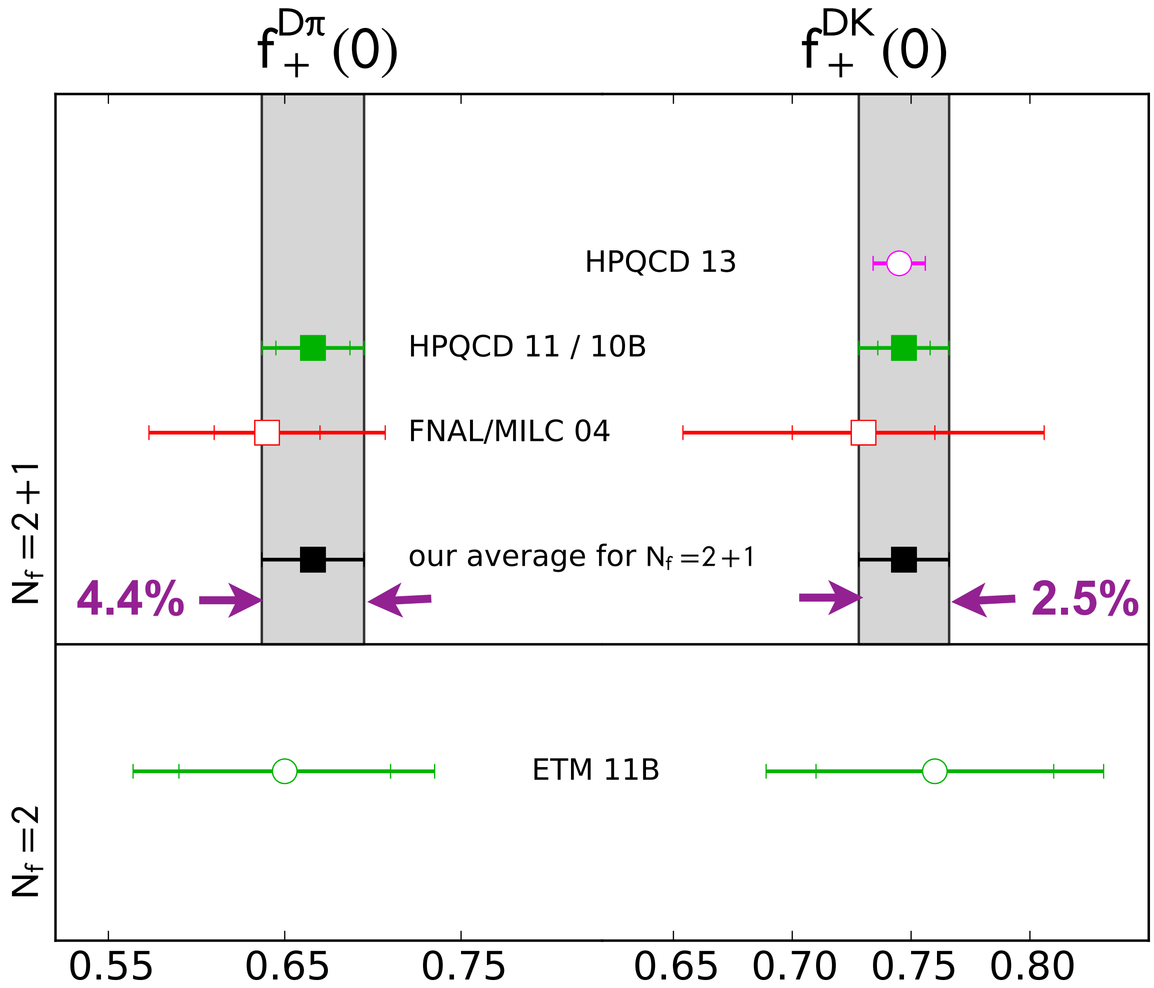
example: $D \to K\ell\nu$



$$\frac{d\Gamma(D \to K\ell\nu)}{dq^2} = (\text{known})\,|\mathbf{p}_K|^3\,|V_{cs}|^2\,\left|f_+^{D\to K}(q^2)\right|^2$$

- ⋆ HFAG average for $f_+(0)|V_{cs(d)}|$: 0.6% (2.1%)
- ⋆ experimental average neglects Coulomb correction in neutral meson decay ~1%
- ⋆ use shape to test LQCD and improve CKM determination

Form factors for $D \to K(\pi)\ell\nu$ & $V_{cs(d)}$



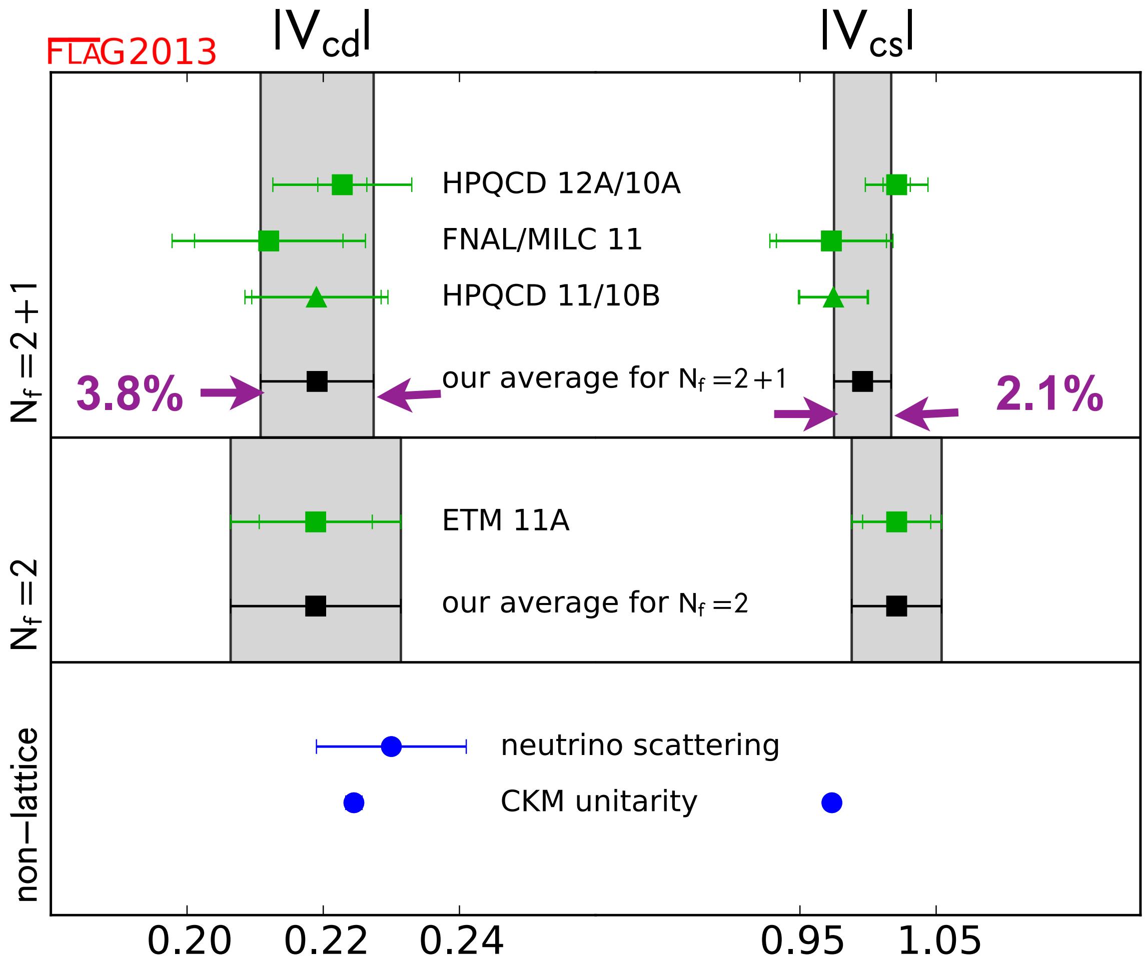
New results (shown in magenta) **not** included in FLAG-2 averages.

small errors due to
- ✦ highly improved action (HISQ)
- ✦ absolutely normalized current

improvement due to
- ✦ adding shape

new results coming soon
- ✦ ETM
- ✦ FNAL/MILC

Implications for V_{cd} & V_{cs}



- B meson physics
  - leptonic

    $$f_B, f_{B_s}, f_{B_s}/f_B$$
  - semileptonic heavy to light

    $$B \to \pi \ell \nu \ \& \ V_{ub} \quad B_s \to K \ell \nu \quad B \to K(\pi) \ell^+ \ell^-$$
  - B to D or $D*$ decays

    $$B \to D^{(*)} \ell \nu \ \& \ V_{cb}$$

    $$B_s \to D_s \ell \nu$$

    $$B \to D \tau \nu$$

B and B_s meson decay constants

experimental measurement suffer from helicity suppression

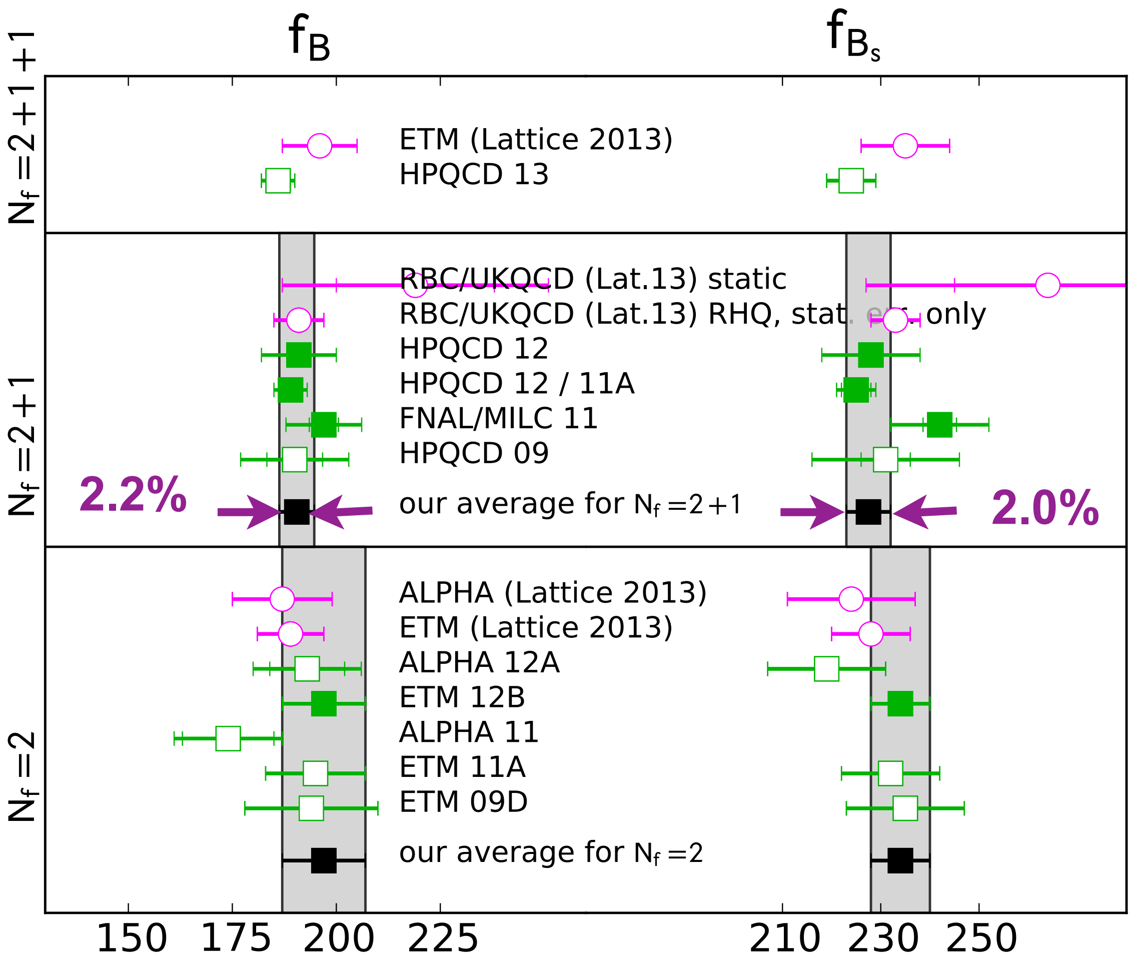
$$B^+ \to \tau^+ \nu_\tau$$

Experimental average for branching fraction: 25%

$$B_s \to \mu^+ \mu^-$$

Experimental average for branching fraction: 24%

B and B_s meson decay constants

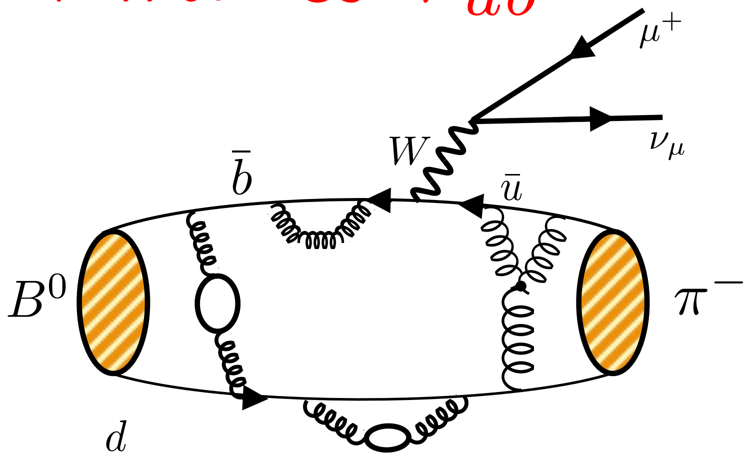


New results (shown in magenta) **not** included in FLAG-2 averages.

small errors due to
✦ highly improved action (heavy HISQ method)
✦ absolutely normalized current

HPQCD 13:
✦ physical mass ensembles
✦ NRQCD-HISQ perturbative matching error dominates

Form factor for $B \to \pi \ell \nu$ & V_{ub}

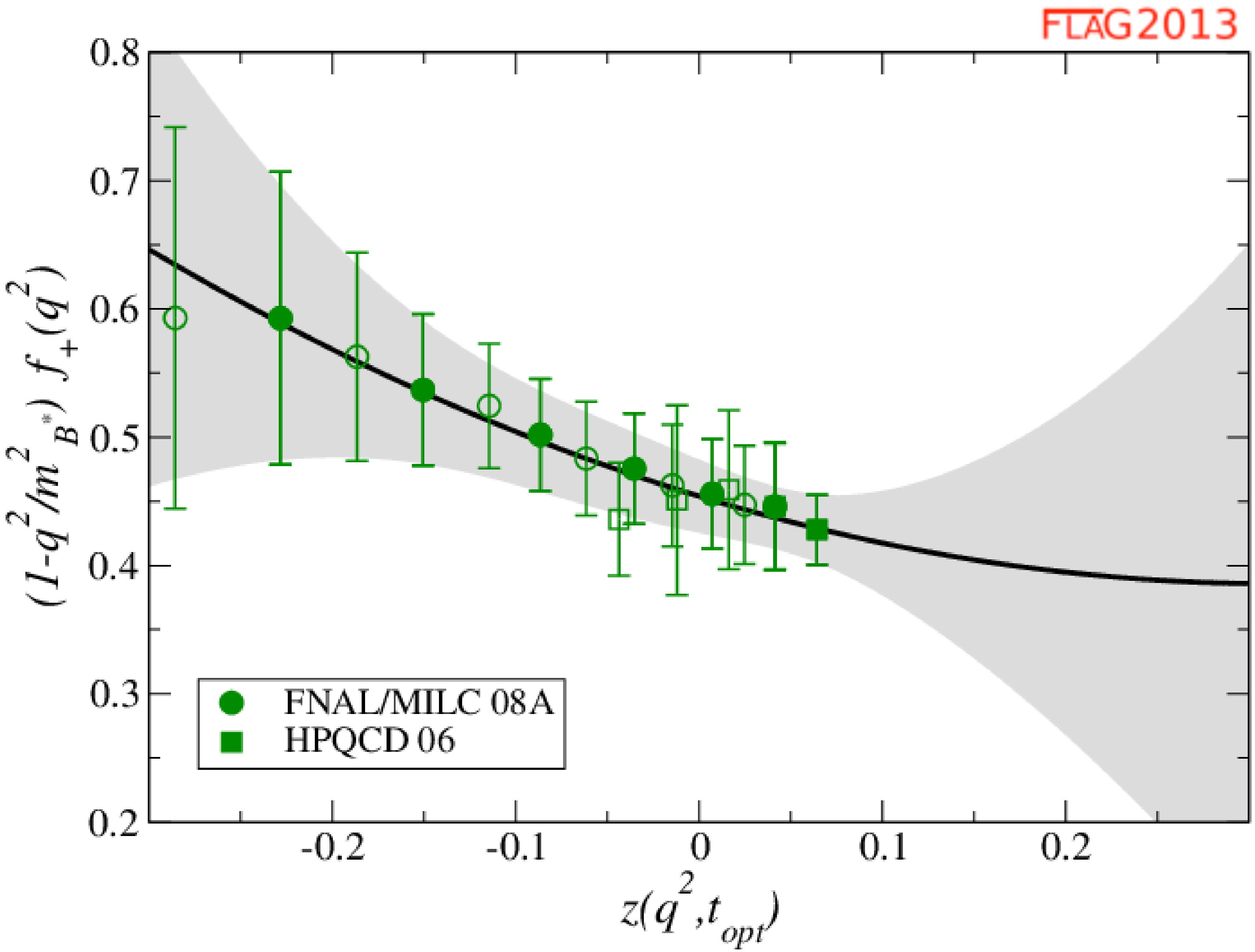


$B \to \pi \ell \nu$

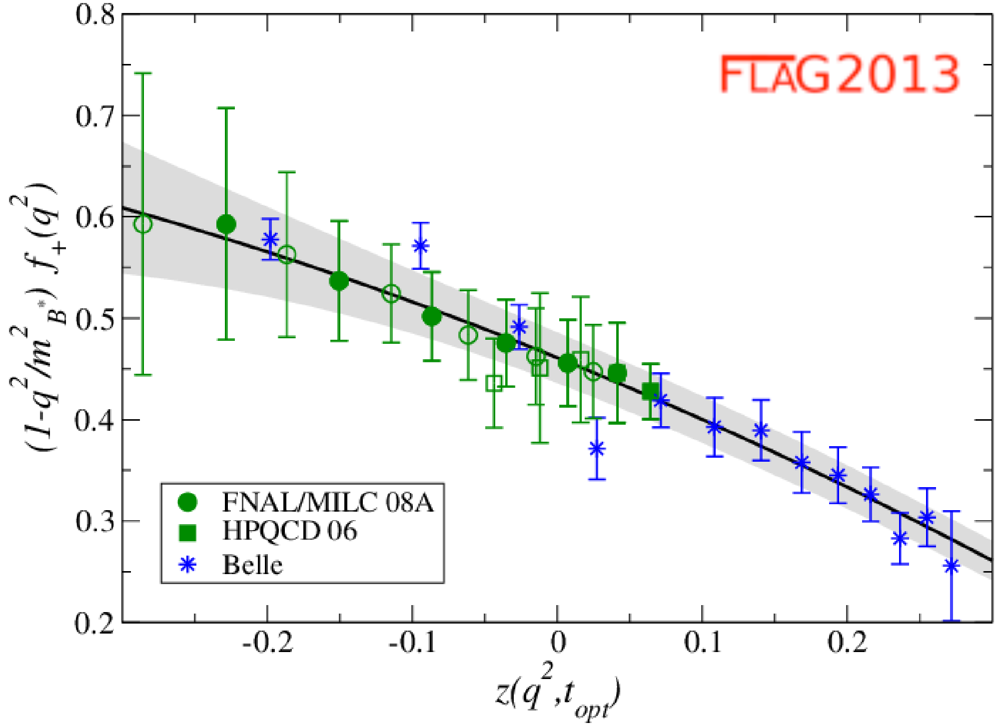
$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2} = (\text{known}) \times |V_{ub}|^2 \times \left| f_+(q^2) \right|^2$$

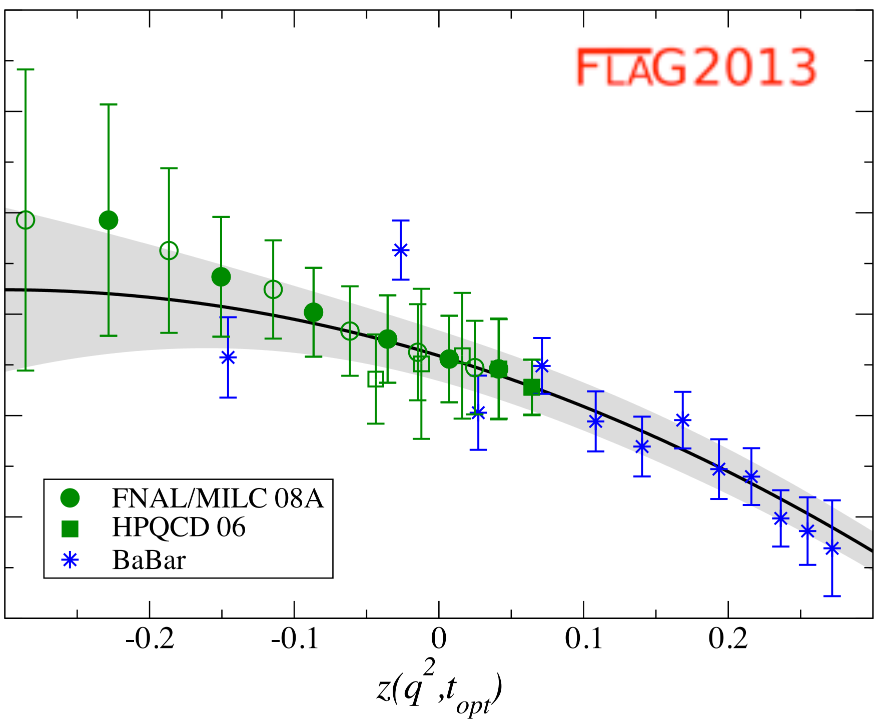
★ shape for semileptonic B decays:
use z-expansion for model-independent parameterization of q^2 dependence

Form factor for $B \to \pi \ell \nu$ & V_{ub}



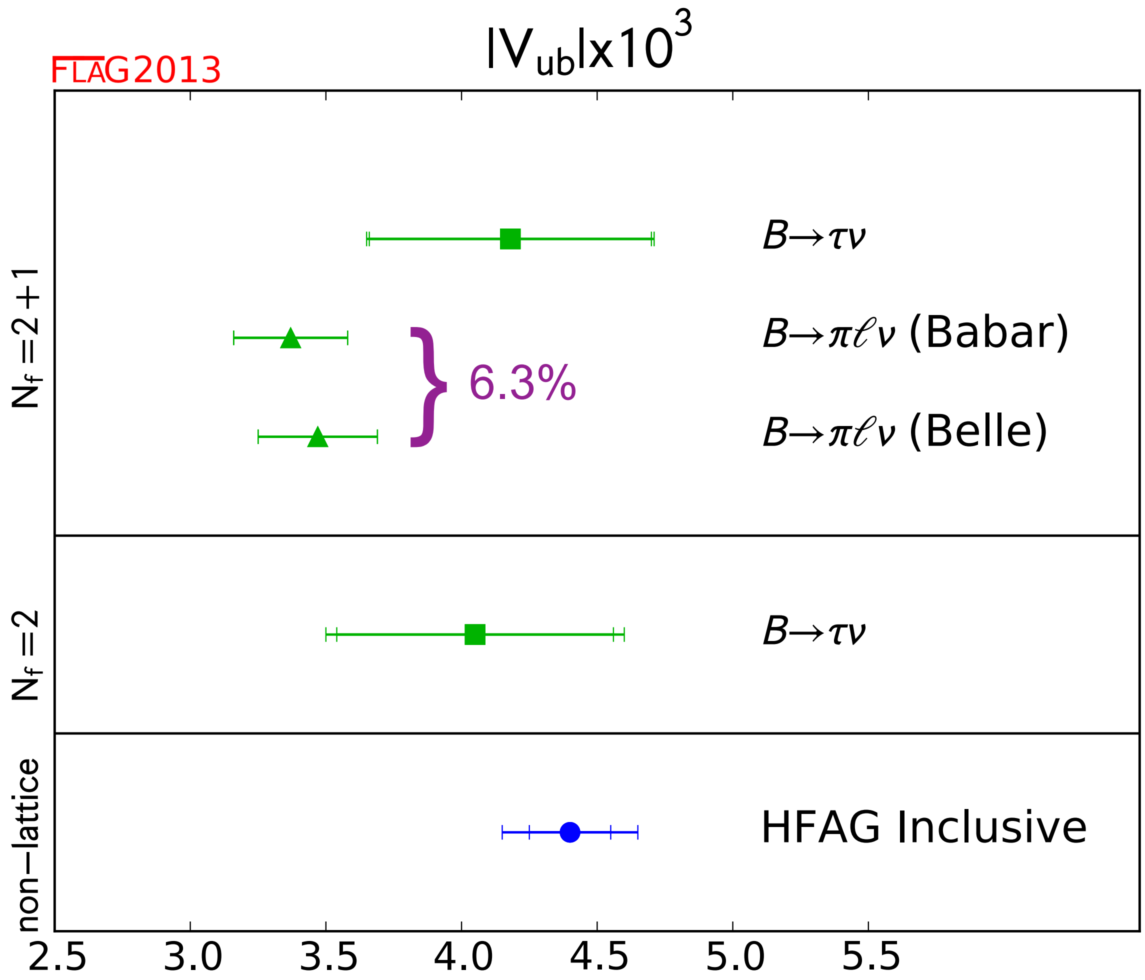
Form factor for $B \to \pi \ell \nu$ & V_{ub}





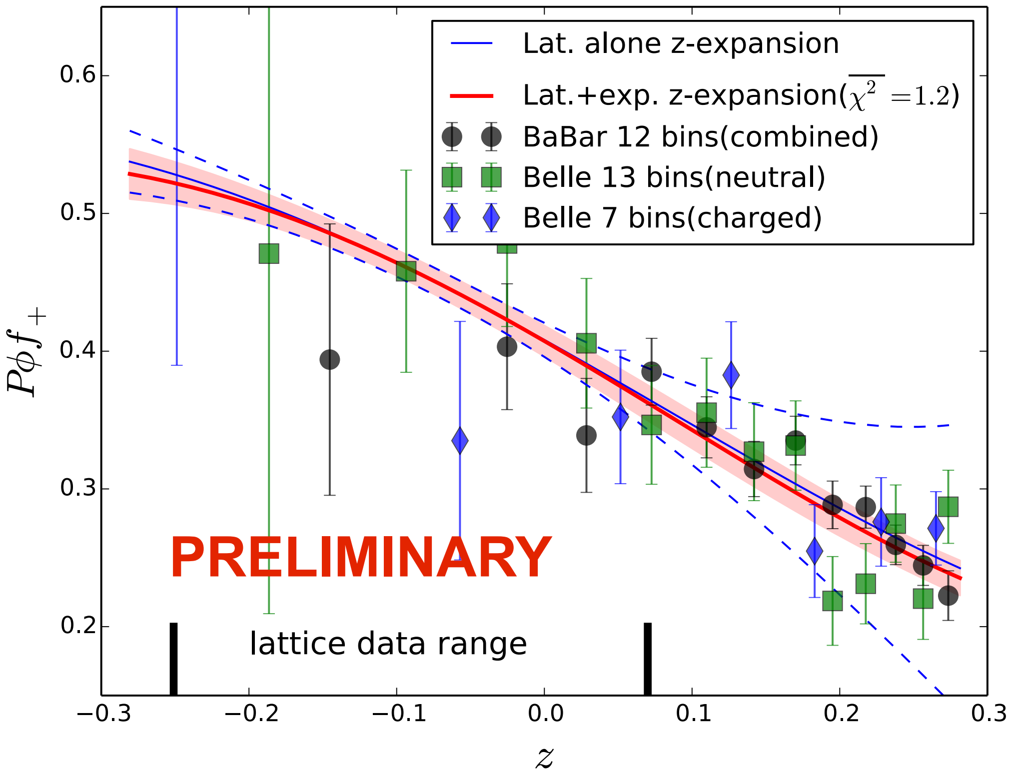
Determine V_{ub} from combined fit.

Implications for V_{ub}



Form factor for $B \to \pi\ell\nu$ & V_{ub}

D. Du (FNAL/MILC) @ Lattice 2013



- **blind analysis**

- $N_f = 2+1$ (Asqtad)
- 4 a's, 12 ensembles
- Fermilab b quarks

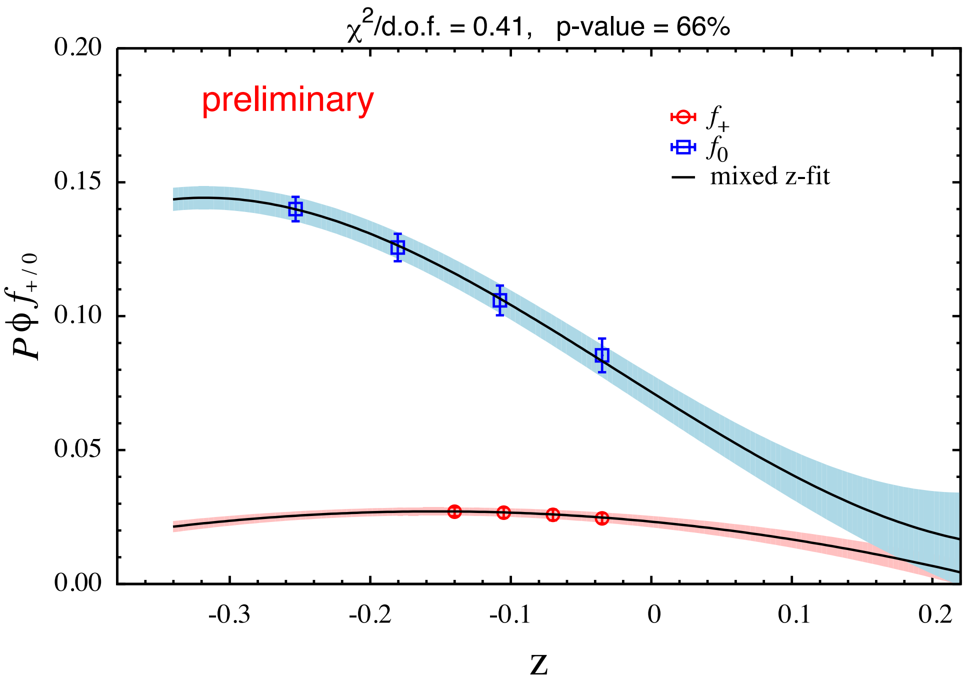
- new functional method for z-expansion fit after chiral extrapolation.

- systematic error analysis in progress.

also in progress: Y. Liu (FNAL/MILC) $B_s \to K\ell\nu$ & V_{ub}

Form factor for $B \to \pi\ell\nu$ & V_{ub}

T. Kawanai (RBC/UKQCD) @ Lattice 2013



- $N_f = 2+1$ (DWF)
- 2 a's, 5 ensembles
- RHQ b quarks

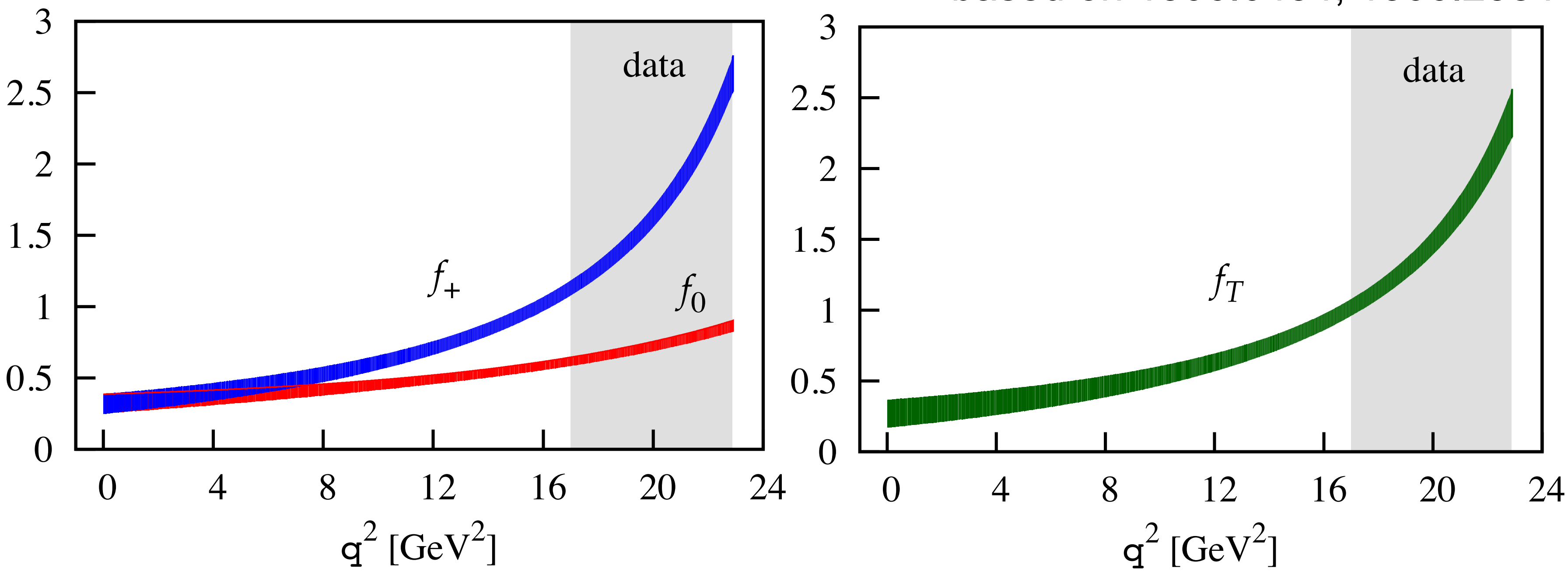
- systematic error analysis in progress.

also: recent work by HPQCD (C. Bouchard @ Lattice 2013) using NRQCD-HISQ quarks

Form factors for $B \to K\ell^+\ell^-$

C. Bouchard (HPQCD) @ Lattice 2013

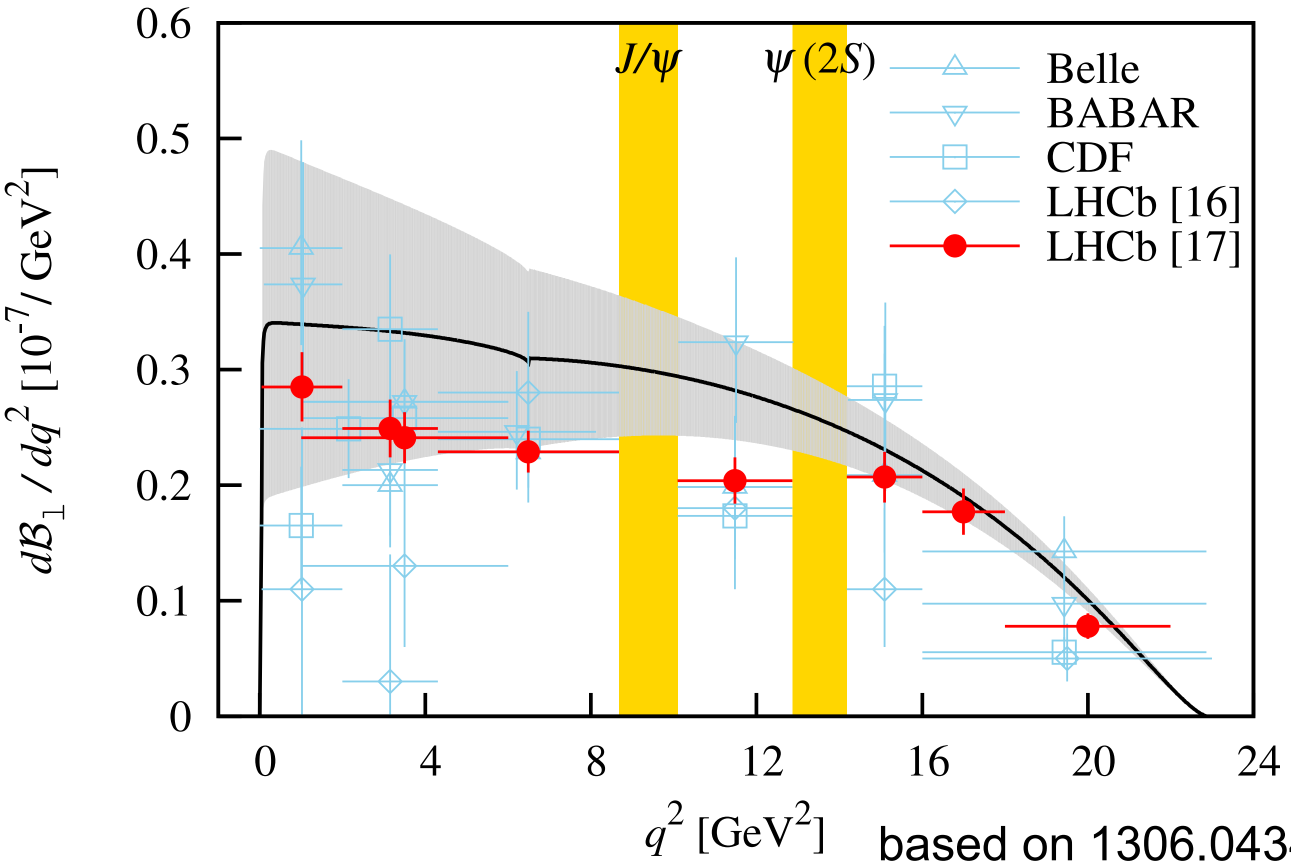
based on 1306.0434, 1306.2384



also in progress: FNAL/MILC (R. Zhou @ Lattice 2013)
using Fermilab b quarks

Form factors for $B \to K\ell^+\ell^-$

SM theory compared to experiment
(courtesy of C. Bouchard)



based on 1306.0434, 1306.2384

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

$$\frac{d\Gamma(B \to D^* \ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{1/2} |\mathcal{F}(\omega)|^2$$

$$\frac{d\Gamma(B \to D\ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{3/2} |\mathcal{G}(\omega)|^2$$

at zero recoil (HFAG 2011):

$$B \to D^*\ell\nu : |V_{cb}|\mathcal{F}(1) = (35.90 \pm 0.45) \times 10^{-3}$$

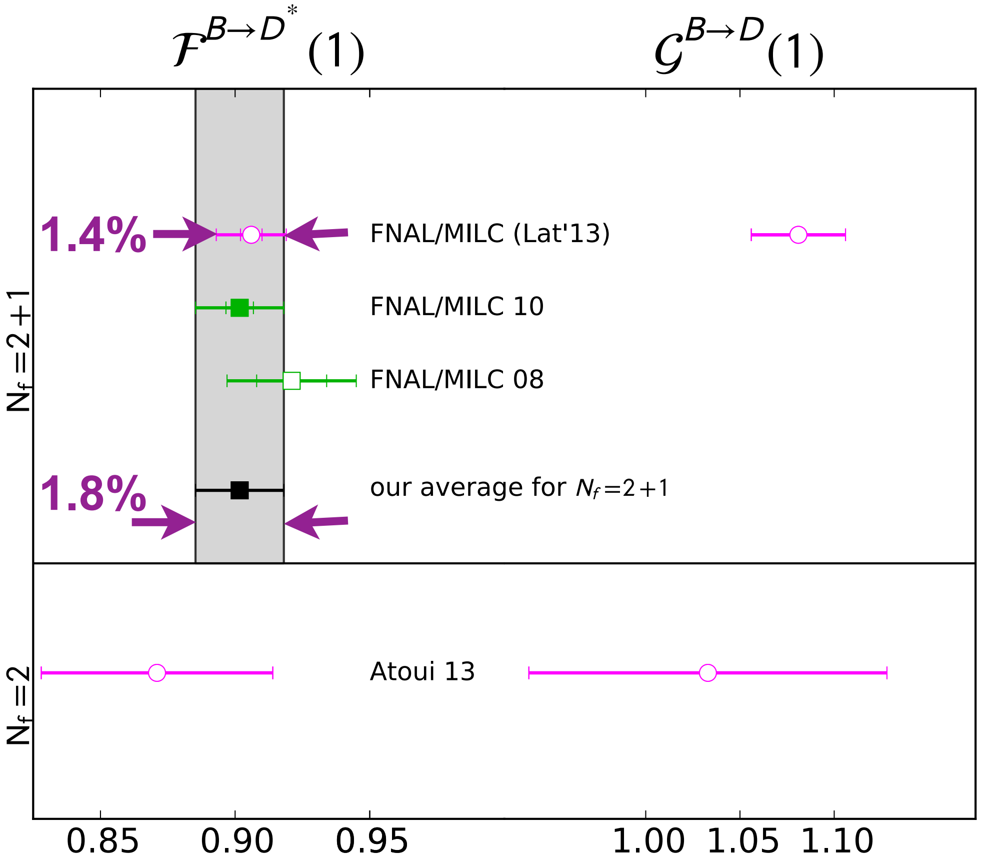
$$B \to D\ell\nu : |V_{cb}|\mathcal{G}(1) = (42.6 \pm 1.5) \times 10^{-3}$$

⇒ need form-factors at non-zero recoil for V_{cb} determination from $B \to D\ell\nu$

Note: the experimental average doesn't include Coulomb correction (~1%) for the neutral meson decay

Saturday, March 8, 14

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}



New results (shown in magenta) **not** included in FLAG-2 averages.

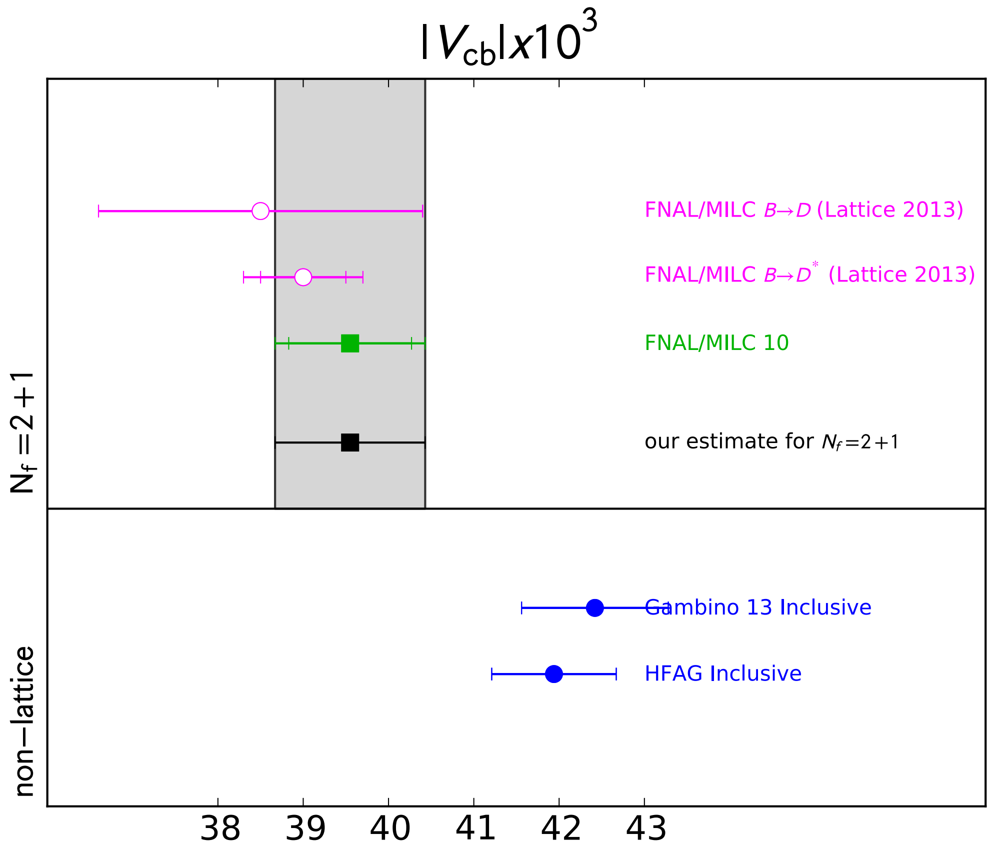
FNAL/MILC:
small errors due to
- ✦ use of ratios
- ✦ 2013:
  5 a's, 12 ensembles

✦ new results by Orsay group using ETM ratio method

✦ work in progress:
  HPQCD (NRQCD-HISQ)
  Bailey (OK action)

Also recent work on $B_s \to D_s^{(*)}$ form factors

Implications for V_{cb}



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FNAL/MILC 2013 (arXiv:1403.0635):

- ✦ estimate of Coulomb correction included, adds 0.5% error
- ✦ LQCD error commensurate with experiment

other interesting quantities

- $B_s \to D_s \ell\nu / B \to D\ell\nu$ & $B_s \to \mu^+\mu-$ (Fleischer et al, arXiv:1004.3984):

$$\frac{f_s}{f_d} = 0.0743 \times \frac{\tau_{B^0}}{\tau_{B^0_s}} \times \left[\frac{\epsilon_{DK}}{\epsilon_{D_s\pi}} \frac{N_{D_s\pi}}{N_{DK}}\right] \times \frac{1}{\mathcal{N}_a \mathcal{N}_F} \qquad \mathcal{N}_F = \left[\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_K^2)}\right]^2$$

form factor ratio calculated in lattice QCD

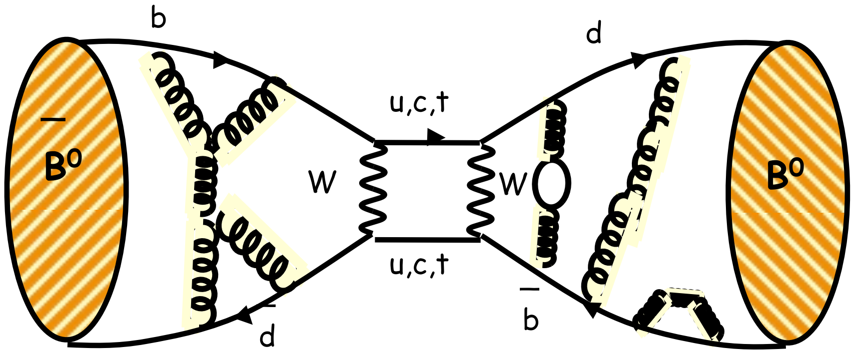
- $R(D) = \mathrm{Br}(B \to D\tau\nu)/\mathrm{Br}(B \to D\ell\nu)$
  measured by BaBar, observed tension with the SM
  depends on scalar form factor
  calculated in lattice QCD

- Neutral meson mixing
  - B mesons $f_B\sqrt{B_B}, f_{B_s}\sqrt{B_{B_s}}, \xi$
  - D mesons

neutral B, and B_s meson mixing

example:

$B_d^0 - \overline{B_d^0}$mixing



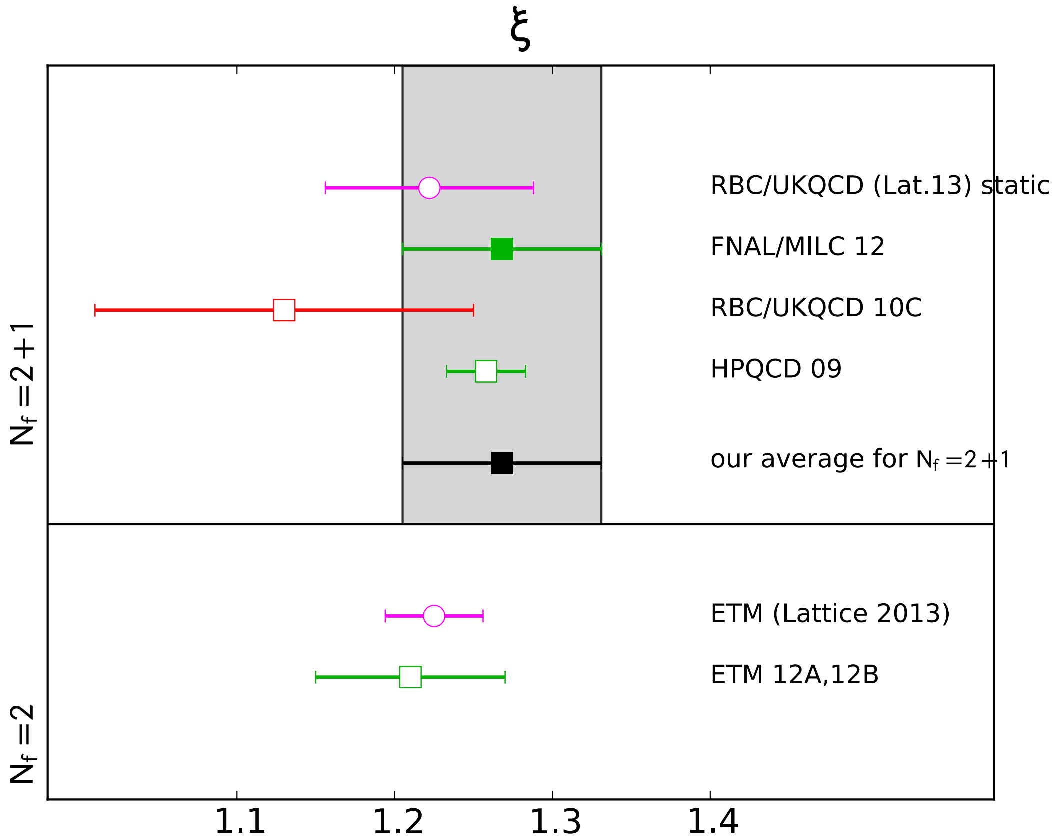
$$\Delta M_d = (\text{known}) \times |V_{td}^* V_{tb}|^2 \times \langle \overline{B^0} | \mathcal{O}_{\Delta B=2} | B^0 \rangle \rightarrow f_{B_d}^2 B_{B_d}$$

also:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{m_{B_s}}{m_{B\,d}} \times \left| \frac{V_{ts}}{V_{td}} \right|^2 \times \xi^2 \quad \text{with} \quad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

- many groups also calculate BSM mixing parameters $\langle \mathcal{O}_{1-5} \rangle$

B and B_s meson mixing parameters



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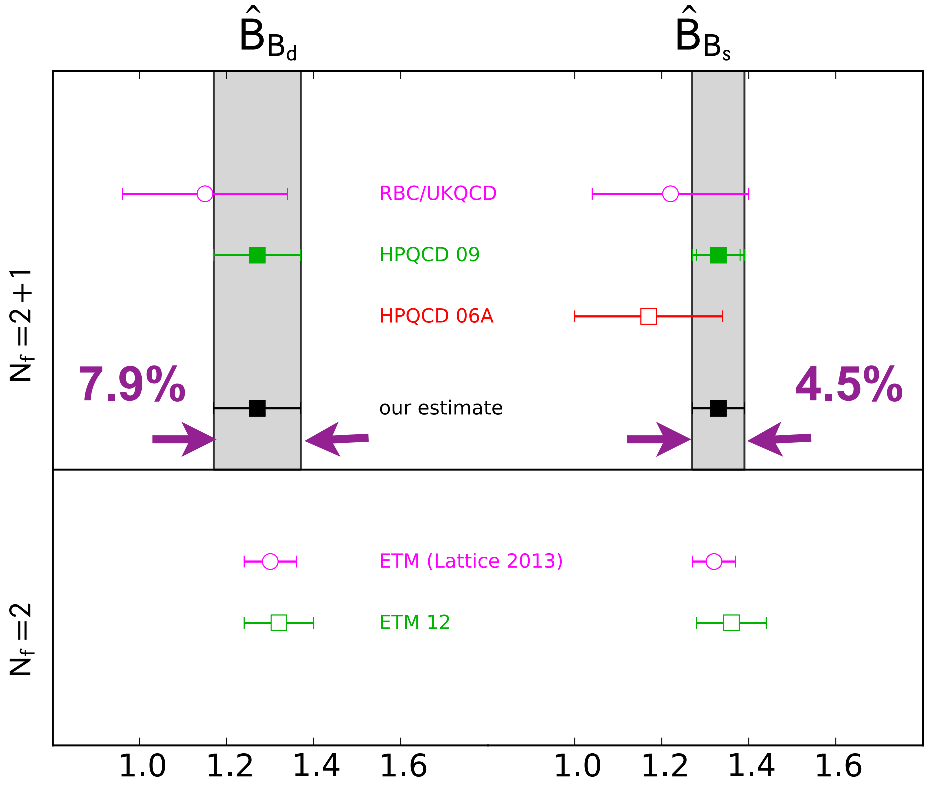
B and B_s meson mixing parameters



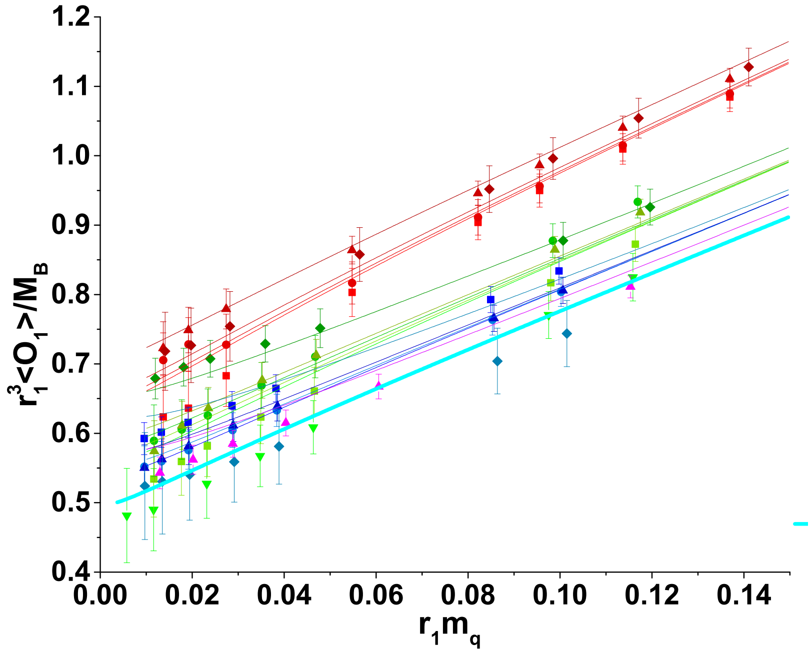
New results (shown in magenta) **not** included in FLAG-2 averages.

✦ new results coming soon

FNAL/MILC
ETM
HPQCD

B and B_s meson mixing parameters

J. Chang (FNAL/MILC) @ Lattice 2013

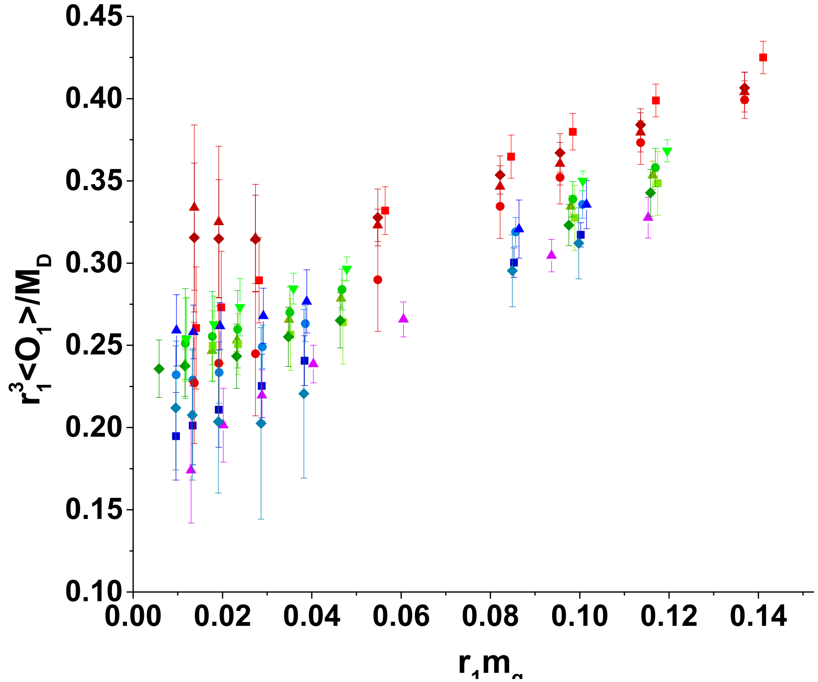


FNAL/MILC:
- ✦ 4 a's, 14 ensembles
- ✦ Fermilab b quarks
- ✦ systematic error analysis in progress

D meson mixing parameters

matrix elements of local operators only

J. Chang (FNAL/MILC) @ Lattice 2013



FNAL/MILC:
✦ 4 a's, 14 ensembles
✦ Fermilab charm quarks
✦ systematic error analysis in progress

ETM (Lattice 2013):
✦ new results with N_f=2+1+1 and N_f=2
✦ using tmWilson charm quarks

Conclusions & Outlook

- three groups have already generated ensembles with **light sea quark masses at their physical values**

  ⇒ expect to see an increasing number of physics results with these

  and an increasing number of such ensembles

- light quark methods for charm: HISQ, tmWilson, NP Wilson, DWF, ...

  ⇒ high precision

- heavy quark methods for b: NRQCD, HQET, Fermilab, RHQ, Tsukuba, heavy HISQ, ETM ratio method, ...

  look for consistency between results with different methods

- If discretization/truncation/matching errors dominate, gain from physical mass ensembles is less apparent

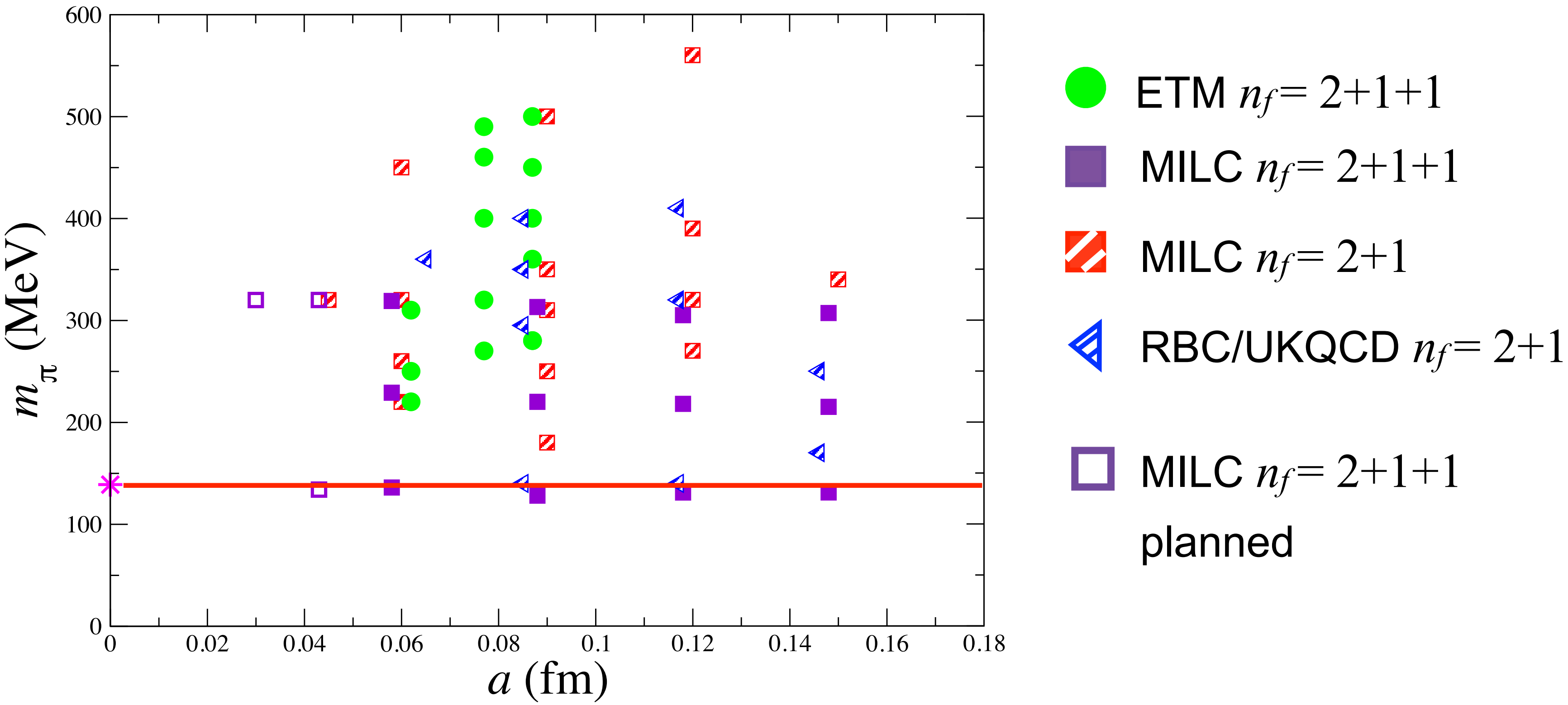
  heavy HISQ, ETM ratio method look promising

- averages: FLAG-2 ⇒ use as inputs to UT fits

- expand LQCD calculations to weak decays of heavy baryons (in progress)

  vector meson final states (in progress)

Conclusions & Outlook



Summary

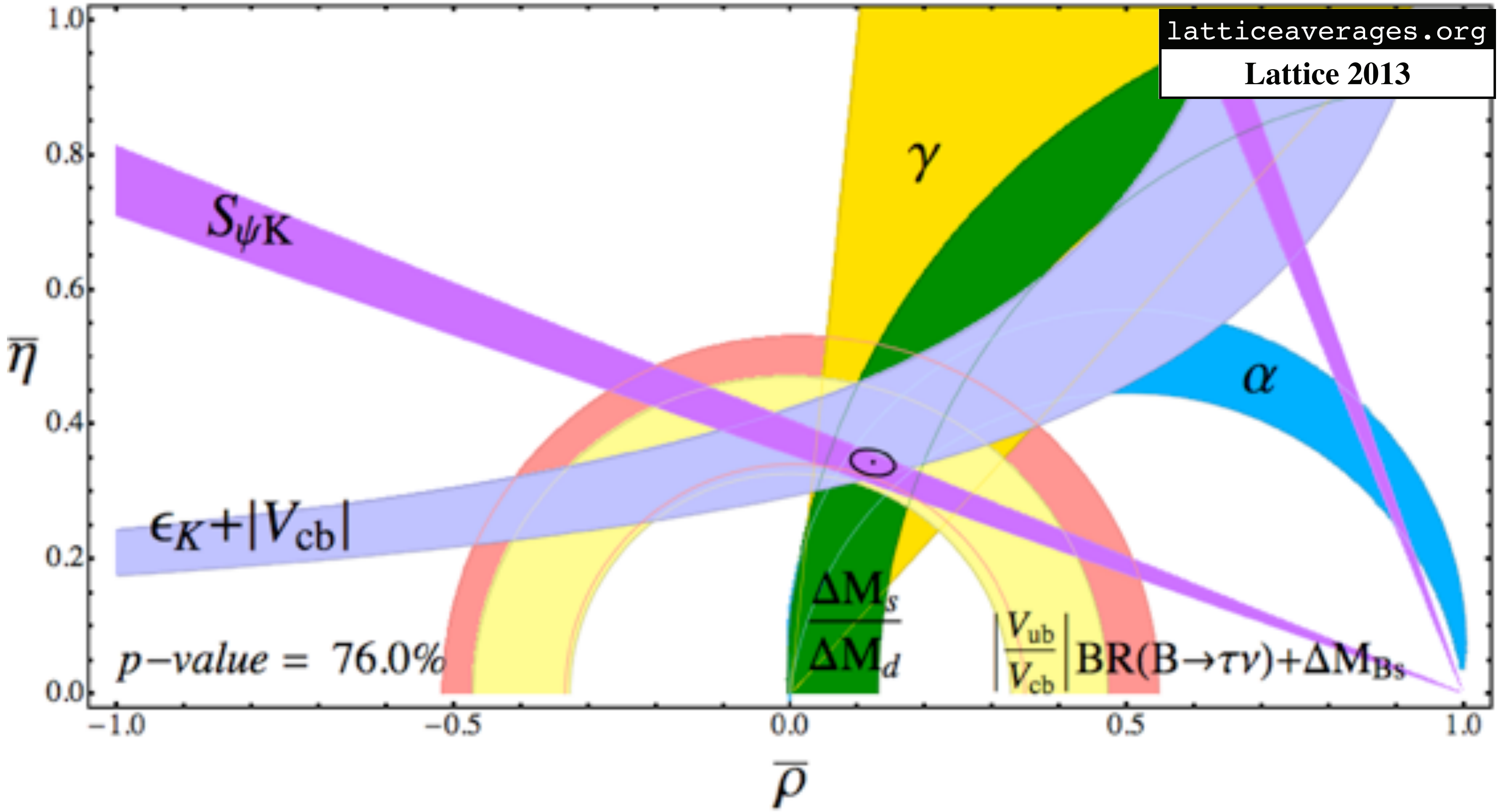
Quark Flavor Physics Working Group (arXiv:1311.1076)

Quantity	CKM element	Present expt. error	2007 forecast lattice error	Present lattice error	2018 lattice error
f_K/f_π	$\|V_{us}\|$	0.2%	0.5%	0.4%	0.15%
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	–	0.4%	0.2%
f_D	$\|V_{cd}\|$	4.3%	5%	~~2%~~ 0.5%	$< 1\%$
f_{D_s}	$\|V_{cs}\|$	2.1%	5%	~~2%~~ 0.5%	$< 1\%$
$D \to \pi\ell\nu$	$\|V_{cd}\|$	2.6%	–	4.4%	2%
$D \to K\ell\nu$	$\|V_{cs}\|$	1.1%	–	2.5%	1%
$B \to D^*\ell\nu$	$\|V_{cb}\|$	1.3%	–	~~1.8%~~ 1.4%	$< 1\%$
$B \to \pi\ell\nu$	$\|V_{ub}\|$	4.1%	–	8.7% → ~4%	2%
f_B	$\|V_{ub}\|$	9%	–	2.5%	$< 1\%$
ξ	$\|V_{ts}/V_{td}\|$	0.4%	2–4%	4%	$< 1\%$
Δm_s	$\|V_{ts}V_{tb}\|^2$	0.24%	7–12%	11%	5%
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$

Table 6. *History, status and future of selected lattice-QCD calculations needed for the determination of CKM matrix elements. 2007 forecasts are from Ref. [112]. Most present lattice results are taken from latticeaverages.org [113]. The quantity ξ is $f_{B_s}\sqrt{B_{B_s}}/(f_B\sqrt{B_B})$.*

Conclusions: UT fit

Enrico Lunghi (LLV)



Conclusions & Outlook

- Sub leading effects:
  - Isospin: leading order correction included via tuning light valence quarks
    (can also include EM isospin corrections)
    effects due to the degenerate sea ~ NNLO in ChPT
  - errors can be further reduced:
    simulations with 1+1+1+1 sea quarks (nondegenerate)
    add QED
  - radiative corrections:
    are already relevant for heavy quark physics ~0.5%
    not straightforward
  - include charm quark in sea ✓

Backup slides

Motivation

time line: quark flavor experiments

a rich history and exciting future!

great discovery potential!

⋮

Project X

~ later ORKA

~ 2015 Belle II, LHC upgrade
NA62
KOTO

~ now LHCb, BaBar, Belle, BES III

CDF, D0, CLEO-c, KLOE, ...

past ARGUS, CLEO, NA48, KTeV, BNL kaon experiments,...

Motivation: $B_s \to \mu^+\mu^-$

S. Hansmann-Menzemer @ EPS 2013

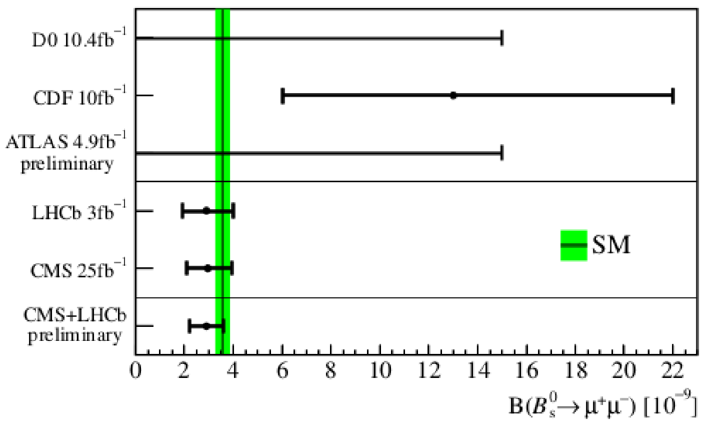
Combined LHCb + CMS Result

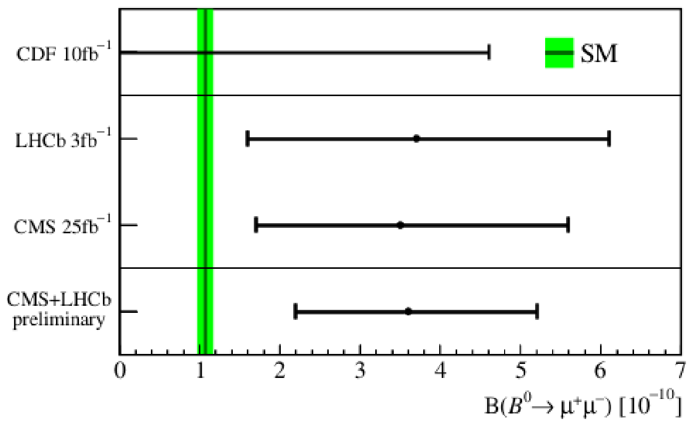
new @ EPS2013

Observation:

BR($B_s \to \mu^+\mu^-$) = (2.9 $\pm$ 0.7) $\times$ 10^{-9}

BR($B^0 \to \mu^+\mu^-$) = 3.6$^{+1.6}_{-1.4}$ $\times$ 10^{-10}

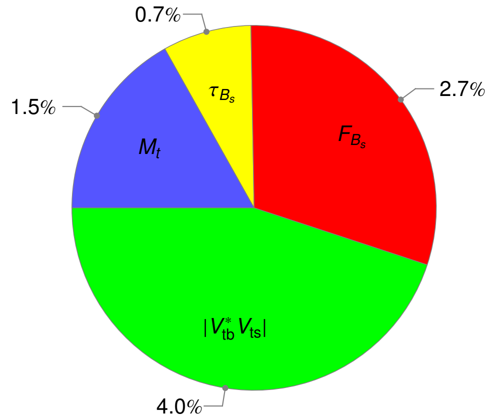




LHCb-CONF-2013-012, CMS-PAS-BPH-13-007

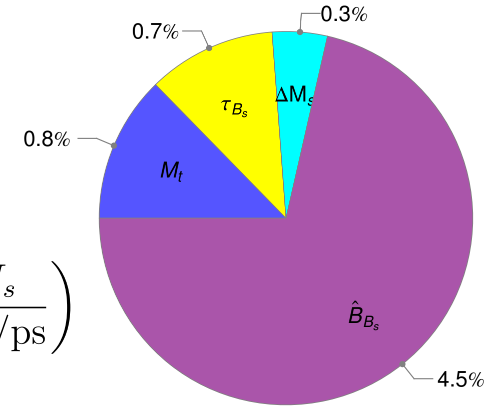
Motivation: $B_s \to \mu^+\mu^-$

Standard Model prediction: Buras, et al, arXiv:1303.3820



$$\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{SM}} = 3.25 \times 10^{-9} \left(\frac{M_t}{173.2\,\mathrm{GeV}}\right)^{3.07} \left(\frac{F_{B_s}}{225\,\mathrm{MeV}}\right)^2 \left(\frac{\tau_{B_s}}{1.500\mathrm{ps}}\right) \left|\frac{V_{tb}^* V_{ts}}{0.0405}\right|^2 .$$

uses f_{B_s} from HPQCD 13

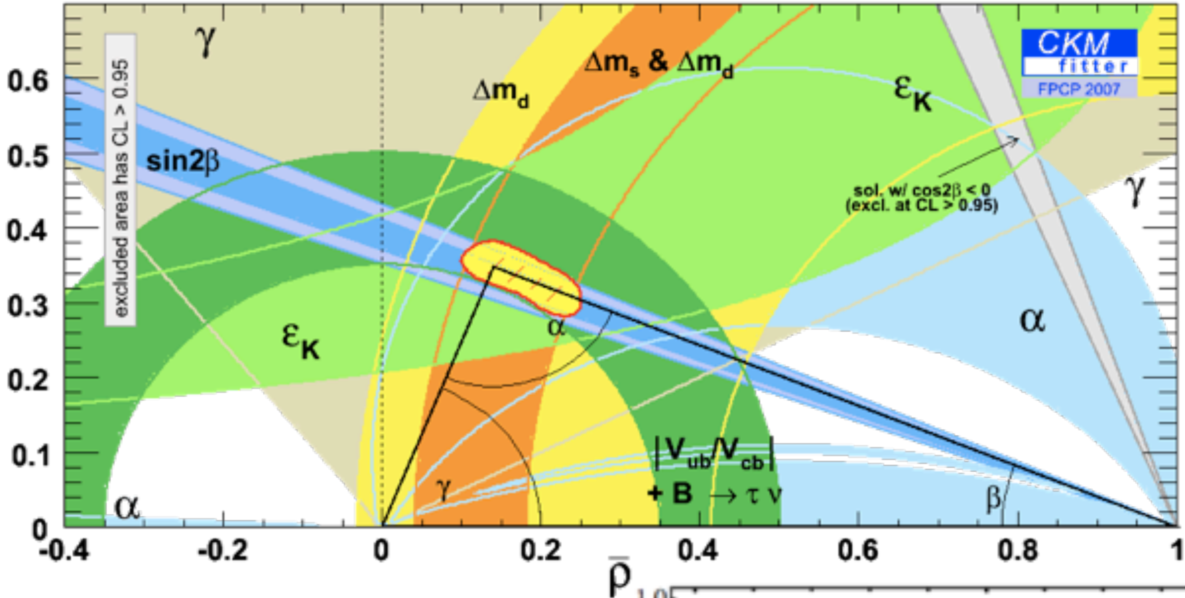


$$\mathrm{BR}(B_s \to \mu^+\mu^-)_{\mathrm{SM}} = 3.38 \times 10^{-9} \left(\frac{M_t}{173.2\,\mathrm{GeV}}\right)^{1.6} \left(\frac{\tau_{B_s}}{1.500\mathrm{ps}}\right) \left(\frac{1.33}{\hat{B}_{B_s}}\right) \left(\frac{\Delta M_s}{17.72/\mathrm{ps}}\right)$$

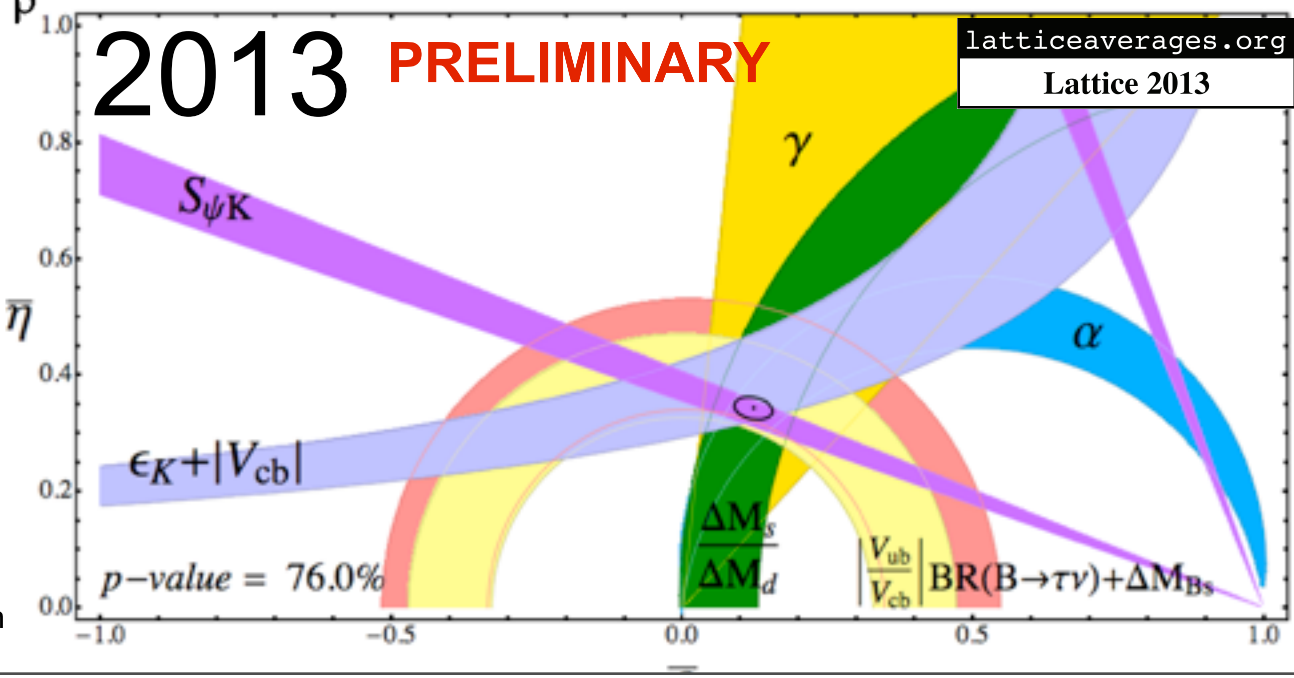
uses $\hat{B}_{B_s}$ from HPQCD 09

Motivation: CKM Unitarity Triangle

2007



Enrico Lunghi



new results reported
at Lattice 2013
not included

A. El-Kh

summary: inputs for the UT fits

Enrico Lunghi

latticeaverages.org
Lattice 2013

$\mathrm{BR}(B \to \tau\nu) = (1.12 \pm 0.27) \times 10^{-4}$	$S_{\psi K_S} = 0.668 \pm 0.023$
$\Delta m_{B_d} = (0.508 \pm 0.004)\ \mathrm{ps}^{-1}$	$\eta_1 = 1.87 \pm 0.76$
$\Delta m_{B_s} = (17.78 \pm 0.12)\ \mathrm{ps}^{-1}$	$\eta_2 = 0.5765 \pm 0.0065$
$m_{t,\mathrm{pole}} = (173.5 \pm 1.0)\ \mathrm{GeV}$	$\eta_3 = 0.496 \pm 0.047$
$m_c(m_c) = (1.273 \pm 0.006)\ \mathrm{GeV}$	$\eta_B = 0.551 \pm 0.007$
$\alpha = (89.5 \pm 4.3)^\circ$	$\gamma = (66 \pm 12)^\circ$ [CKMfitter]
$\varepsilon_K = (2.229 \pm 0.012) \times 10^{-3}$	$\lambda = 0.2253 \pm 0.0009$
$\hat{B}_K = 0.766 \pm 0.010$	$f_K = (156.3 \pm 0.9)\ \mathrm{MeV}$
$\kappa_\varepsilon = 0.94 \pm 0.02$	$f_B = (190.5 \pm 4.2)\ \mathrm{MeV}$
$f_{B_s}\sqrt{\hat{B}_{B_s}} = (266 \pm 18)\ \mathrm{MeV}$	$\xi \equiv f_{B_s}\sqrt{\hat{B}_s}/(f_{B_d}\sqrt{\hat{B}_d}) = 1.268 \pm 0.063$
$\lvert V_{ub}\rvert_{\mathrm{incl}} = (4.34 \pm 0.16^{+0.15}_{-0.22}) \times 10^{-3}$	$\lvert V_{cb}\rvert_{\mathrm{incl}} = (41.68 \pm 0.44 \pm 0.09 \pm 0.58) \times 10^{-3}$
$\lvert V_{ub}\rvert_{\mathrm{excl}} = (3.41 \pm 0.20) \times 10^{-3}$	$\lvert V_{cb}\rvert_{\mathrm{excl}} = (39.55 \pm 0.72 \pm 0.50) \times 10^{-3}$
$\lvert V_{ub}\rvert_{\mathrm{avg}} = (3.77 \pm 0.44) \times 10^{-3}$	$\lvert V_{cb}\rvert_{\mathrm{avg}} = (40.8 \pm 1.0) \times 10^{-3}$

summary: UT fit output

Enrico Lunghi (LLV)

latticeaverages.org
Lattice 2013

The predictions from all other information when the direct determination of the quantity is removed from fit are

$$|V_{ub}| = (3.49 \pm 0.13) \times 10^{-3} \quad (0.6\,\sigma) \tag{10}$$

$$S_{\psi K} = 0.757 \pm 0.050 \quad (1.7\,\sigma) \tag{11}$$

$$|V_{cb}| = (42.48 \pm 1.1) \times 10^{-3} \quad (1.1\,\sigma) \tag{12}$$

$$\widehat{B}_K = 0.855 \pm 0.11 \quad (0.80\,\sigma) \tag{13}$$

$$f_{B_d}\sqrt{\widehat{B}_d} = (206.3 \pm 5.4)\ \mathrm{MeV} \quad (0.61\,\sigma) \tag{14}$$

$$\mathrm{BR}(B \to \tau\nu) = (0.776 \pm 0.065) \times 10^{-4} \quad (1.2\,\sigma) \tag{15}$$

$$\begin{cases} f_{B_d} = (228. \pm 29.)\ \mathrm{MeV} \quad (1.3\,\sigma) & \text{complete fit} \\ f_{B_d} = (208.2 \pm 31.)\ \mathrm{MeV} \quad (0.56\,\sigma) & \text{without using } S_{\psi K} \end{cases} \tag{16}$$